

من فضلك ابدأ حل التحليلية من اليسار ( the cover ) والتكامل من اليمين

**Question 1 [12 marks] Complete the following**

- (1) The conic section  $x^2 + 4y^2 - 2x - 3 = 0$  is named ....., its center located at ....., and the length of the latus rectum equals .....
- (2) Given the parabola  $x^2 - 2x + 8y - 15 = 0$ . The vertex is ....., the equation of its axis is ....., and the coordinates of its focus is .....
- (3) Given the two circles  $x^2 + y^2 = 4$  and  $x^2 + y^2 - 6y + k = 0$ . The two circles
  - cut orthogonally if  $k = \dots\dots\dots$
  - touch internally if  $k = \dots\dots\dots$
  - have a radical axis  $y = 3$  if  $k = \dots\dots\dots$
- (4) If a line  $L$  passes through the point  $(2, 3, 4)$  and has the following properties :
  - intersects the plane  $Ax + By + z = 6$  at the point  $(1, 2, 2)$
  - perpendicular to the plane  $2x - By + Cz = 1$
 then  $A = \dots\dots\dots$ ,  $B = \dots\dots\dots$ , and  $C = \dots\dots\dots$

**Question 2 [13 marks]**

**(a) [ 8 marks ] True or False: (justify the answer. Zero credit for no explanation)**

- (1) The eccentricity of the conic  $x^2 - 4y^2 - 2x - 16y + 1 = 0$  equals 0.6
- (2) The area bounded by the region  $x^2 + y^2 \leq 4$  equals the area bounded by the region  $x^2 + y^2 - 6x + 5 \leq 0$ .
- (3) The three points  $(-1, 3, 2)$ ,  $(-4, 2, -2)$  and  $(5, 5, 10)$  are collinear
- (4) The two lines  $L_1$  and  $L_2$  are coplanar, where

$$L_1 : x = \frac{y+3}{2} = \frac{z+1}{3} \quad \text{and} \quad L_2 : \frac{x-3}{2} = y = \frac{z-1}{-1}$$

**(b) [5 marks]** Given the plane  $\pi: 2x - y + z + 3 = 0$ , and the point  $P = (1, 3, 4)$ . The point  $P$  does not lie in the plane  $\pi$ . Find:

- (1) The equation of a line  $L$  passing through  $P$  and perpendicular to the plane  $\pi$
- (2) The coordinates of the point  $Q$  that lies on the line  $L$  and in the plane  $\pi$  as well
- (3) The coordinates of the point  $S$  which is the reflection of  $P$  in the plane  $\pi$



### Question 3 13 marks

A) Complete each of the following

- I) The method of evaluating the integral  $\int \frac{\cos x}{\sin^3 x - 1} dx$  is .....
- II) The substitution used to evaluate the integral  $\int \frac{x^2}{(4+x^2)^{5/2}} dx$  is ..... or .....
- III) If  $\int \tan^2 x \cosh(f(x)) dx = \sinh(f(x)) + c$  then  $f(x) = \dots\dots\dots$
- IV)  $\int_{-\pi}^{\pi} |\sin x| dx = \int_0^{-\pi} \dots\dots\dots dx + \int_0^{\pi} \dots\dots\dots dx$

B) Determine whether  $\int_0^{\pi/2} \sec x dx$  converges or diverges.

C) Find  $\int \frac{\tan^{-1} x + \ln(\tan^{-1} x)}{1+x^2} dx$

D) For the curve  $f(x) = \int_x^1 u \sqrt{\cosh u} du$ ,

- I) Find  $f(1)$  and  $f(-1)$
- II) Find the slope of the tangent to  $f(x)$  at  $x = 0$ .

### Question 4 12 marks

A) I) Find the equation of the horizontal line bisecting the area in the fourth quadrant bounded by the curve  $y = \ln x$  and the coordinate axes.

II) Find the volume of the solid generated by revolving this area about  $y - axis$ .

B) The equation  $S = 2\pi \int_1^2 \cosh^{-1} x \sqrt{1 + (f'(x))^2} dx$  represents the area of a surface generated by revolving the arc  $y = f(x)$ ,  $1 \leq x \leq 2$  about  $x - axis$ .

Find the length of this arc.

C) Using the definition of Laplace transform, find the value of  $L\{\sin x\}$

$\int \frac{dx}{1+x^2}$	$\int \frac{dx}{1-x^2}$	$\int \frac{dx}{x^2-1}$	$\int \sinh x dx$	$\cosh x$	$\int \sin x dx$	$-\cos x$
$\tan^{-1} x$	$\tanh^{-1} x$	$-\coth^{-1} x$	$\int \cosh x dx$	$\sinh x$	$\int \cos x dx$	$\sin x$
$\int \frac{dx}{\sqrt{1-x^2}}$	$\int \frac{dx}{\sqrt{1+x^2}}$	$\int \frac{dx}{\sqrt{x^2-1}}$	$\int \operatorname{sech}^2 x dx$	$\tanh x$	$\int \sec^2 x dx$	$\tan x$
$\sin^{-1} x$	$\sinh^{-1} x$	$\cosh^{-1} x$	$\int \operatorname{csch}^2 x dx$	$-\coth x$	$\int \csc^2 x dx$	$-\cot x$
$\int \frac{dx}{x\sqrt{x^2-1}}$	$\int \frac{dx}{x\sqrt{1-x^2}}$	$\int \frac{dx}{x\sqrt{1+x^2}}$	$\int \operatorname{sech} x \tanh x dx$	$-\operatorname{sech} x$	$\int \sec x \tan x dx$	$\sec x$
$\sec^{-1} x$	$-\operatorname{sech}^{-1} x$	$-\operatorname{csch}^{-1} x$	$\int \operatorname{csch} x \coth x dx$	$-\operatorname{csch} x$	$\int \csc x \cot x dx$	$-\csc x$

Best wishes

Dr. Reda Abdo Dr. Samir Shamseldeen

$\int e^x dx$	$e^x$	$\int x^n dx$	$x^{n+1}/(n+1)$
$\int a^x dx$	$a^x/\ln a$	$\int dx$	$x$
$\int \frac{1}{x} dx$	$\ln x$	$\int \frac{1}{\sqrt{x}} dx$	$2\sqrt{x}$



### 3<sup>rd</sup> Question [14 marks]

(a) Graph and write the equation of parabola which has focus at  $(0, -2)$  and a directrix  $x = 2$ . [2 marks]

(b) Find the Distance between the point  $Q(2, -3, 1)$  and the line  $\frac{x-3}{2} = \frac{y+5}{1} = \frac{z-1}{-2}$ . [2 marks]

(c) For the sphere  $S: x^2 + y^2 + z^2 = 25$  and the plane  $\varphi: 2x + y + 2z - 9 = 0$ . Find the center  $P_0$  and the radius  $r$  of the circle of intersection of the plane and the sphere. [3 marks]

(d) i) Graph and write the name of each of the following surfaces [7 marks]

$$S_1: z = x^2.$$

$$S_2: 2x + 3y = 6.$$

$$S_3: \frac{(x-1)^2}{4} - \frac{(y-3)^2}{9} + \frac{(z-4)^2}{4} = 1$$

ii) Describe the traces in planes  $x = 3$  and  $y = 3$  with the surface  $S_3$ .

iii) Describe the trace in plane  $x = 0$  with the surface  $S_2$ .

### 4<sup>th</sup> Question [11 marks]

(a) Complete the following: [3 marks]

i)  $\int \frac{\cot x}{\sqrt{1+\ln \sin x}} dx = \dots\dots\dots$

ii) If  $f(x)$  is an even function and  $\int_0^1 f(x) dx = 5$  &  $\int_2^1 f(x) dx = 4$

then  $\int_{-1}^1 f(x) dx = \dots\dots\dots$ ,  $\int_{-2}^2 (\sin x + f(x)) dx = \dots\dots\dots$ ,

$\int_0^2 f(x) dx = \dots\dots\dots$  and  $\int_{-2}^2 (\sin x f(x)) dx = \dots\dots\dots$

(b) Evaluate the following integrals [4 marks]

$$I_1 = \int \frac{2x-1}{\sqrt{x^2-2x+2}} dx$$

$$I_2 = \int \frac{6}{x^3+x^2-2x} dx$$

(b) (i) Find the length of the arc of the curve [2 marks]

$$x = \cos(t) + t \sin(t), y = \sin(t) - t \cos(t), 0 \leq t \leq \pi$$

(ii) Find the integral that represents the area of the surface generated by rotating the arc about

1) Line  $x = 1$ .

2) Line  $y = 2$ . [2 marks]

With my best wishes

Dr.Ayman Gomaa

**Question (1) [13 marks]**

a) [7 marks] For the following conic equation

$$x^2 + 4y^2 - 6x - 8y = 3$$

Complete the following table

(من فضلك في كراسة الإجابة ضع الأجابات بالترتيب وفي جدول)

1	Equation in standard form and name	
2	Center and vertices	
3	Eccentricity	
4	Foci	
5	Directrices	
6	Graph of the conic	
7	The area of a triangle $OAB$ , where $O$ is the origin, $A$ and $B$ are the conic vertices	

b) [6 marks] Given the surface equation

$$x^2 + 2z^2 = y - 1$$

- 1) State the name of the surface and determine its (vertex/centre) and its axis
- 2) Describe the traces in the  $xy$  plane,  $xz$  plane,  $y = 1$  and  $y = 5$  planes
- 3) Graph the surface

**Question (2) [12 marks]**

a) [6 marks] Evaluate the following integrals (if exist)

$$1) \int_1^2 \sqrt{2x - x^2} \, dx$$

$$2) \int_0^\infty \frac{1}{\sqrt{x}} \, dx$$

b) [6 marks] A region is bounded by

$$y = e^x, \quad y = 1 \quad \text{and} \quad x = 2$$

- 1) Determine the area of that region
- 2) Determine the volume resulting from revolving that region about
  - i)  $x$ -axis
  - ii)  $y$ -axis